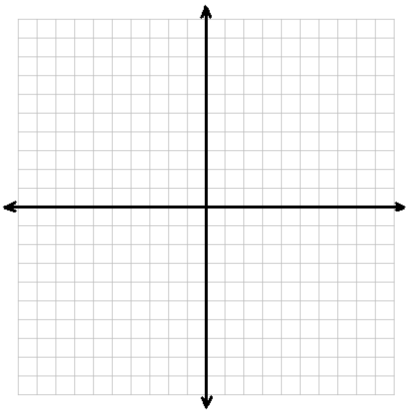
The Definite Integral (Sections 5.4 and 5.5)

Warm –up: Two Ways to Calculate Area Under a Curve

1. Graph $y = \frac{x}{2} + 1$ and calculate the area under it (using geometry formulas) over [0,3].



2. Now estimate the area by using the right endpoint rectangle method.

a) n = 3

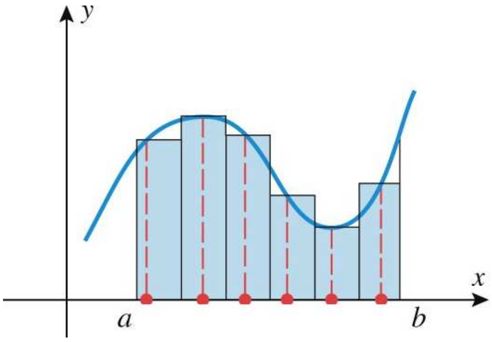
i	x_i	$f(x_i)$	A_i
0			
1			
2			
3			

b) n = 12

i	x_i	$f(x_i)$	A_i
0			
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

3. Which approximation is closer to the exact area found in #1?

This leads to the general conclusion that the approximation of area from the rectangle method gets closer and closer to the exact area as ______________________________.



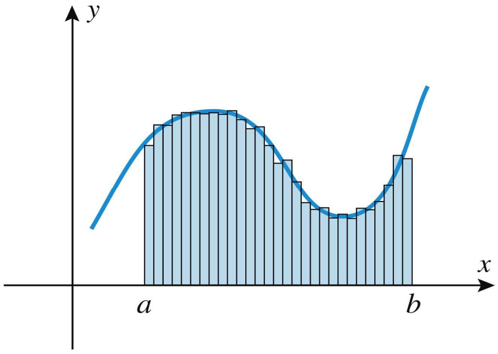


Figure 5.4.5

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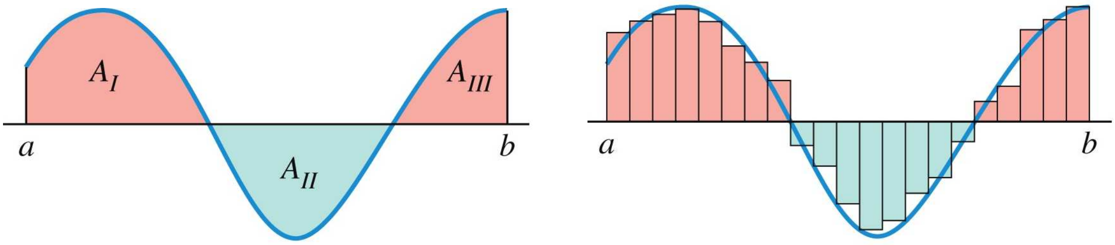
The Definition of Area as a Limit

Definition (Area as a Limit)

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

Net Signed Area



(a) (b)

Areas below the x-axis are considered negative. Why? ________________________________

____.

Definition of the Definite Integral

Definite Integral Definition

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x = \int_a^b f(x)dx$$

The Definite Integral (Sections 5.4 and 5.5)

Properties of the Definite Integral

1. $\int_a^a f(x)dx = 0$

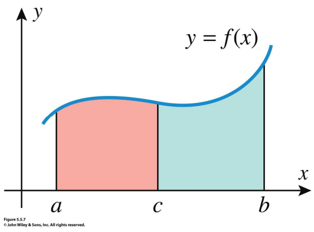
2. $\int_b^a f(x)dx = -\int_a^b f(x)dx$

3. $\int_a^b cf(x)dx = c\int_a^b f(x)dx$

4. $\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

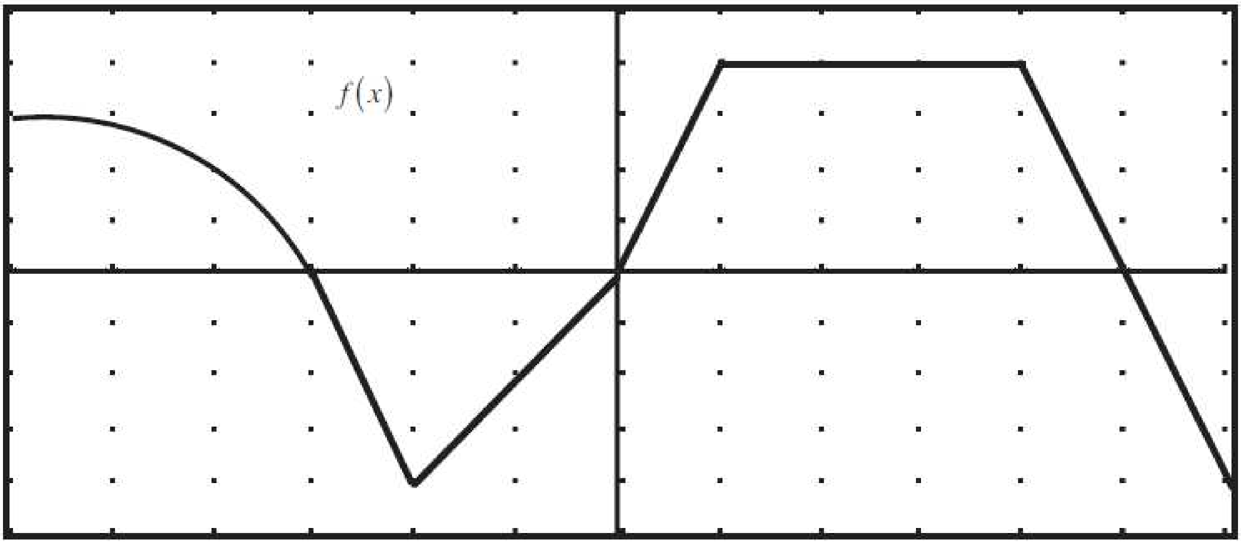
5. $\int_a^b [f(x) - g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx$

6. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ (if c is on $[a, b]$)



The Definite Integral (Sections 5.4 and 5.5)

Classwork/Homework



1. $\int_0^1 f(x)\,dx$
2. $\int_2^4 f(x)\,dx$
3. $\int_1^4 f(x)\,dx$
4. $\int_5^5 f(x)\,dx$
5. $\int_4^5 f(x)\,dx$
6. $\int_5^6 f(x)\,dx$
7. $\int_4^6 f(x)\,dx$
8. $\int_0^6 f(x)\,dx$
9. $\int_3^2 f(x)\,dx$
10. $\int_5^0 f(x)\,dx$
11. $\int_6^0 f(x)\,dx$
12. $\int_{-3}^0 f(x)\,dx$
13. $\int_0^{-3} f(x)\,dx$
14. $\int_{-3}^2 f(x)\,dx$
15. $\int_4^{-3} f(x)\,dx$
16. $\int_{-6}^{-3} f(x)\,dx$
17. $\int_{-3}^{-6} f(x)\,dx$
18. $\int_{-6}^0 f(x)\,dx$
19. $\int_{-6}^6 f(x)\,dx$
20. $\int_6^{-6} f(x)\,dx$
21. $\left|\int_{-2}^1 f(x)\,dx\right|$
22. $\int_{-2}^1 |f(x)|\,dx$
23. $\int_{-2}^1 |-f(x)|\,dx$
24. $\int_{-6}^6 |f(x)|\,dx$

Suppose that $\int_0^2 f(x)\,dx = 2$, $\int_1^2 f(x)\,dx = -1$, $\int_2^4 f(x)\,dx = 7$, evaluate the following:

25. $\int_1^4 f(x)\,dx$
26. $\int_0^4 3f(x)\,dx$
27. $\int_0^1 f(x)\,dx$
28. $\int_0^1 f(x+1)\,dx$
29. $\int_0^2 (f(x)+3)\,dx$
30. $\int_2^4 f(x-2)\,dx$